NAME
Yao Zhou
(China)

ADVISOR
O.Univ.-Prof. Dipl.Ing.Dr.
Ulrich Langer

INSTITUTE
RICAM
JOHANN-RADON-INSTITUTE
FOR COMPUTATIONAL AND APPLIED MATHEMATICS

UNIVERSITY OF APPLIED SCIENCES OBERÖSTERREICH

JOHANNES KEPLER UNIVERSITY LINZ

softwarepark hagenberg

ISI International Studies in Informatics
HAGENBERG

Fourier Analysis and Local Fourier Analysis for Multigrid Methods

Fourier Analysis
Local Fourier Analysis
Multigrid Methods
Linear Algebraic Systems
Optimal Control Problem
Convergence Factor

The aim of this master thesis is to apply Fourier analysis and Local Fourier Analysis (LFA) to calculate the exact convergence factor of two-grid methods for the solution of linear algebraic systems arising in different applications. We first consider the one-dimensional Poisson problem. Both Fourier analysis and LFA are used to derive two-grid convergence factors. Differences between these two analysis tools are illustrated. Then we study a saddle-point problem stemming from an optimal control problem. The smoothing factor for the optimal control problem is derived by Fourier Analysis for a special preconditioner. Moreover, the robustness of the preconditioner is shown by the theoretical result as well as a numerical experiment.

Introduction

Multigrid methods are among the fastest numerical algorithms for solving linear systems arising from a discretization technique such as the finite element method or the finite difference method. The convergence speed does not deteriorate when the discretization is refined, whereas classical iterative methods slow down for decreasing mesh size.

Multigrid algorithms are composed of four elements:
(1) smoothing of rough error parts via classical iterative methods;
(2) approximation of smooth errors on a coarser grid;
(3) recursive application of (1) and (2) on a sequence of coarser and coarser grids;
(4) nested iteration for producing good initial guesses.

Approach

Considering multigrid algorithms, there is an enormous degree of freedom in choosing the algorithmic components. The practical very important question is how to choose individual multigrid components for concrete situations.

There are two related approaches in treating this question: Fourier Analysis and LFA. They are considered as the main analysis tools to obtain quantitative convergence estimates and to optimize multigrid components like smoothers or intergrid transfer operators.