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Symbolic Computation for Parametrized Wavelets in SAGE

Wavelets | Gröbner Bases | SAGE | Parametrization | Discrete Moments | Orthonormal | Biorthogonal

We parametrize wavelets in both orthonormal and biorthogonal cases with symbolic methods, in particular Gröbner bases methods, in an open source computer algebra system SAGE. New parametrized filter coefficients and more general parametrization methods are discussed.

Wavelets

Wavelets are useful in many areas, e.g. signal process, image compression and etc.

In general, any algorithm relying on wavelet uses corresponding filter coefficients but not the wavelet function itself.

The wavelet design problem amounts to the filter design problem.

Gröbner Bases

We use Gröbner Bases to solve multivariate non-linear equation systems in this master thesis.

Parametrization

$$\begin{cases} \sum_{k=0}^{N} h_k = 2 \quad \text{(Normalization)} \\ \sum_{k=0}^{N} (-1)^k h_k = 0 \quad \text{(First sum rule)} \\ \sum_{k=0}^{N} (-1)^{n-k} h_k (n-k)^l = 0,\ for\ 0 < l < p \ \text{(Vanishing moments)} \\ \sum_{k=0}^{N} h_k h_{k+2i} = 2\delta_{0,l},\ for\ i = 1, ..., (N-1)/2 \ \text{(Orthonormal)} \\ \sum_{k=0}^{N} h_k k^n = m_n \quad \text{(Discrete moments)} \end{cases}$$

Motivation

We want to obtain more degree of freedom to make wavelet filter satisfies more additional conditions.

Idea

We parametrize wavelet by giving up higher vanishing moments conditions and using discrete moments instead as parameters.

Contribution

We parametrize orthonormal and biorthogonal wavelets with discrete moments and compute the parametried filter coefficients with symbolic methods, in particular Groebner bases method, and discuss a more general case in parametrizing biorthogonal wavelets.

We give all corresponding worksheets in SAGE for the computation.

We analyze the details of the parametrization procedures and results.

We also give SAGE worksheets for some wavelets experiments in a symbolic way, e.g. symbolic discrete wavelet transform.